PREFERENCE INTENSITY IN MCDM WHEN AN ADDITIVE UTILITY FUNCTION REPRESENTS DM PREFERENCES

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We propose a new method for ranking alternatives in multicriteria decision-making problems when there is imprecision concerning the alternative performances, component utility functions and weights. We assume decision maker's preferences are represented by an additive multiattribute utility function, in which weights can be modeled by independent normal variables, fuzzy numbers, value intervals or by an ordinal relation. The approaches are based on dominance measures or exploring the weight space in order to describe which ratings would make each alternative the preferred one. On the one hand, the approaches based on dominance measures compute the minimum utility difference among pairs of alternatives. Then, they compute a measure by which to rank the alternatives. On the other hand, the approaches based on exploring the weight space compute confidence factors describing the reliability of the analysis. These methods are compared using Monte Carlo simulation.

1. Introduction

In multicriteria decision-making, the classical additive multiattribute utility model is considered to be a valid approach in many practical situations[2]. Incorporating imprecision concerning weights and/or component utilities is one way of extending the model to closely describe real situations: less information is usually available than is needed to determine the best alternative[9]. Sarabando and Dias[7] give a brief overview of approaches within the multiattribute utility theory framework to deal with incomplete information.

A recent approach is to use information about each alternative's intensity dominance, known as *dominance measuring methods*. Ahn and Park[1] were the first to propose a dominance measuring method. This method computes both dominating and dominated measures and then derives a *net dominance*. Net dominance is used as a measure of the strength of preference in the sense that a greater net value is better. Mateos et al.[5,6] propose and compare two alternative methods aimed at improving Ahn and Park's methods. Mateos et al.[5] consider uniformly distributed intervals to model imprecision concerning weights, whereas Mateos et al.[6] consider ordinal relations among attribute weights.

A second approach to deal with imprecision is the Stochastic Multicriteria Acceptability Analysis (*SMAA*)[3]. *SMAA* computes *acceptability indices*, which measure the variety of different preferences that rank each alternative as best. This information can be used to classify the alternatives as more or less acceptable and as unacceptable. However, *SMAA* ignores information about the other ranks. This problem is solved in Lahdelma and Salminen[4] using the SMAA-2 method, which extends the analysis to the sets of weight vectors for any rank from best to worst for each decision alternative.

In this paper, we extend and improve the methods proposed in Mateos et al.[5,6]. Instead of weight intervals for each attribute or ordinal relations among attribute weights, we assume weights can follow independent normal distributions or fuzzy numbers (triangular or trapezoidal). In Section 2 we introduce the extension of dominance measuring methods proposed in Mateos et al.[5,6]. Section 3 reports a simulation study of the methods outlined in previous sections. Finally, some conclusions are considered.

2. A New Preference Intensity Method (*NPIM*)

We consider a set of alternatives $A_1, \dots, A_m$ evaluated on attributes $X_1, \dots, X_n$, a utility function $u_j(x_{ij})$ on the performance x_{ij} of alternative A_i under attribute X_j, a set of weights w_j and the well-known functional form $u(A_i) = \sum_{j=1}^{n} w_j u_j(x_{ij})$, $\forall i$. This represents the utility of each alternative A_i, $\forall i$. The incomplete information about input parameters has been incorporated into the decision-making process: (1) alternative performances under uncertainty and imprecision concerning utility function assessment, $u_j(x_{ij}) \in \boldsymbol{U}_{ij}$; and (2) uncertainty about weights, which can be represented by: (a) ordinal relations, $\boldsymbol{W} = \{w\colon w_1 \geq w_2 \geq \cdots \geq w_n\}$; (b) value intervals, $\boldsymbol{W} = \{w\colon w_j \in [w_j^L, w_j^U], \forall j\}$; (c) intervals for weight ratios, $\boldsymbol{W} = \{w\colon w_j/w_k \in [w_{jk}^L, w_{jk}^U]\}$; (d) linear inequality constraints for weights, $\boldsymbol{W} = \{Aw \leq c\}$; (e) nonlinear inequality constraints for weights, $\boldsymbol{W} = \{g(w) \leq 0\}$; (f) independent normal distributions, $\boldsymbol{W} = \{w\colon w_j \sim N(\mu_j, \sigma_j^2), \forall j\}$; (f) fuzzy numbers, $\boldsymbol{W} = \{\widetilde{w}\colon \widetilde{w}_j, \forall j\}$, where $\widetilde{w}_j$ are triangular or trapezoidal fuzzy numbers.

In this section we propose a method based on the concept of dominance. Given two alternatives A_k and A_l, alternative A_k dominates A_l if $D_{kl} \geq 0$, D_{kl} being the optimum value of the optimization problem:

$$D_{kl} = \min_{\substack{w \in \boldsymbol{W} \\ u_j(x_{kj}) \in \boldsymbol{U}_{kj}, \forall j \\ u_j(x_{lj}) \in \boldsymbol{U}_{lj}, \forall j}} \left\{ u(A_k) - u(A_l) = \sum_{j=1}^{n} w_j \left(u_j(x_{kj}) - u_j(x_{lj}) \right) \right\} \qquad (2)$$

The method is implemented in the following 4 steps.

1. Compute D_{kl} for alternatives A_k and A_l $(\forall k,l)$ following (2) and the intervals $\boldsymbol{I}_{kl} = [D_{kl}, -D_{lk}]$, if uncertainty about weights can be represented by style a), b), c), d) or e), see Section 2.
2. Compute proportions P_{kl} as follows:

a. If uncertainty about weights is represented by style a), b), c), d) or e), see Section 2, then

$$P_{kl} = \begin{cases} d(\boldsymbol{I}_{kl}, 0), & if\ D_{kl} \geq 0 \\ \frac{-D_{lk}+D_{kl}}{-D_{lk}-D_{kl}} d(\boldsymbol{I}_{kl}, 0), & if\ D_{kl} < 0\ and {-D_{lk}} > 0; \\ -d(\boldsymbol{I}_{kl}, 0), & if\ -D_{lk} \leq 0 \end{cases}$$

b. If uncertainty about weights is represented by style f), then

$$P_{kl} = \left(\int_0^\infty f_{kl}(x)dx\right) d(\boldsymbol{I}_{kl}, 0) - \left(1 - \int_0^\infty f_{kl}(x)dx\right) d(\boldsymbol{I}_{kl}, 0)$$

where $f_{kl}(x)$ is the density function of the variable Dkl and $I_{kl} = [\mu - 2\sigma, \mu + 2\sigma]$;

a. If uncertainty about weights is represented by style g), then

$$P_{kl} = \frac{\int_0^\infty f_{kl}(x)dx}{\int_{-\infty}^\infty f_{kl}(x)dx} d(D_{kl}, 0) - \frac{\int_{-\infty}^0 f_{kl}(x)dx}{\int_{-\infty}^\infty f_{kl}(x)dx} d(D_{kl}, 0)$$

where $f_{kl}(x)$ is the membership function of the fuzzy number D_{kl}. Here $d(\cdot,\cdot)$ is a distance measure[8].

3. Compute a preference intensity measure for each alternative A_k: $P_k = \sum_{l=1, l\neq k}^{m} P_{kl}$.
4. Rank alternatives according to the P_k values, where the best (rank 1) is the alternative with greatest P_k and the worst is the alternative with the least P_k.

3. Simulation Study

Let us compare the proposed method (*NPIM*) with the Mateos et al's[5,6] (*PIM*), Ahn and Park's[1] (*AP*), *SMAA*[3] and *SMAA-2*[4] methods.

We propose to carry out a simulation study of the above methods to analyze their performance. The process would be as follows: (1) Randomly generate component utilities for each alternative in each attribute from a uniform distribution in (0,1), leading to an $m \times n$ matrix. Remove dominated alternatives; (2) Generate attribute weights. Note that these weights are the TRUE weights, and the derived ranking of alternatives will be denoted as the TRUE ranking. The resulting weights will sum 1 and be uniformly distributed in the weight space; (3) To derive the corresponding weight intervals, add and subtract the same quantity to precise values, leading to the lower and upper endpoints of the weight intervals. We used the quantities, q, of 0.05, 0.1, 0.15, 0.2 and 0.25 that represent 10%, 20%, 30%, 40% and 50% imprecision, respectively. In other words, $[w_i^L, w_i^U]= [w_i^T-q, w_i^T+q]$. If $w_i^T-q<0$, then $w_i^T-q=0$, and if $w_i^T+q>1$, then $w_i^T+q=1$ is considered. Throughout the simulation process weights will be randomly generated from these weight intervals, $[w_i^T-q, w_i^T+q]$; (4) Compute the ranking of alternatives for each method according to their procedures and compare with the TRUE ranking, computed in step 2. We used two measures of efficacy, *hit ratio* and *rank-order correlation*[1]. The hit ratio is the proportion of all cases in which the method selects the same best alternative as in the TRUE ranking. Rank-order correlation represents how similar the overall structures ranking alternatives are in the TRUE ranking and in the ranking derived from the method. It is calculated using Kendall's τ.

Four different levels of alternatives (m = 3, 5, 7, 10) and five different levels of attributes (n = 3, 5, 7, 10, 15) were considered in order to validate the results output. Also, 20,000 trials were performed for each of the 20 design elements (alternatives × attributes).

Table 1. Average hit ratios.

Alt.	*Atrib.*	*NPIM*	*PIM*	*AP*	*SMAA*	*SMAA-2*
3	3	0.9937	0.9923	0.9813	0.9962	0.9962
	5	0.9878	0.9856	0.9744	0.9900	0.9901
	7	0.9778	0.9765	0.9610	0.9820	0.9824
	10	0.9652	0.9596	0.9459	0.9697	0.9698
	15	0.9300	0.9248	0.9043	0.9469	0.9470
5	3	0.9902	0.9880	0.9697	0.9947	0.9951
	5	0.9815	0.9773	0.9540	0.9856	0.9857
	7	0.9709	0.9645	0.9404	0.9743	0.9752
	10	0.9504	0.9404	0.9163	0.9600	0.9606
	15	0.9110	0.90085	0.8701	0.9271	0.9273
7	3	0.9906	0.9867	0.9667	0.9935	0.9935
	5	0.9768	0.9711	0.9446	0.9814	0.9818
	7	0.9657	0.9549	0.9249	0.9727	0.9732
	10	0.9438	0.9311	0.89985	0.9509	0.9512
	15	0.9025	0.8853	0.8455	0.9081	0.9087
10	3	0.9895	0.9826	0.9603	0.9924	0.9926
	5	0.9753	0.9658	0.9317	0.9790	0.9798
	7	0.9602	0.9431	0.9079	0.9620	0.9631
	10	0.9379	0.9191	0.8807	0.9397	0.9406
	15	0.8944	0.8742	0.8326	0.8921	0.8925

Table 1 exhibits the average hit ratio for each of the 20 design elements when the interval length is 0.1, i.e., the average values of 20,000 trials.

If we consider the average rank-order correlation or other levels of imprecision, the results are very similar.

We can conclude that: (1) The new method and *SMAA-2* are similar, and the difference between them is negligible; (2) The new method is easier to apply than *SMAA-2* because the multidimensional integrals generally have to be computed using numerical techniques in *SMAA-2*; (3) The new method can be applied when uncertainty about weights is represented by fuzzy numbers, whereas *SMAA-2* is not applicable in these cases; (4) All methods return a worse hit ratio when imprecision increases, as is to be expected.

4. Conclusions

In this paper we consider a new method to ranking alternatives when there is imprecision in the DM's preferences. This method is based on dominance measures. This method has the following advantages. First, it is applicable when the uncertainty about the weights of the additive multiattribute utility function can be represented by ordinal relations, value intervals, intervals for weight ratios, linear inequality constraints for weights, independent normal distributions or fuzzy numbers. Second, the method is one of the best considering the proportion of all cases in which the method selects the same best alternative or the rank-order correlation, as two measures of the goodness of the methods. Third, the method operations are very straightforward. The most complicated operation is the optimization problem, whereas integrals have to be solved to compute volumes to apply the *SMAA* or *SMAA-2* methods. Numerical techniques are generally required to compute these integrals. Monte Carlo simulation was used to apply *SMAA* and *SMAA-2*.

A future research line is to analyze new preference intensity measures that consider the centroid values for the imprecision.

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